EXERCISE – II MULTIPLE CORRECT (OBJECTIVE QUESTIONS)

1. Which one of the following is homogeneous function ?

(A) f(x, y) = $\dfrac{x-y}{x^2+y^2}$ (B) $f(x,y)=x^{\frac{1}{3}}.y^{-\frac{2}{3}}\tan^{-1}\dfrac{x}{y}$

(C) f(x, y) = x ($\ell n\sqrt{x^2+y^2}$ – ℓny) + ye$^{x/y}$

(D) f(x, y) = $x\left[\ell n\dfrac{2x^2+y^2}{x}-\ell n(x+y)\right]$ + $y^2\tan\left(\dfrac{x+2y}{3x-y}\right)$

2. The function f(x) satisfying the equation,
$f^2(x) + 4f'(x) . f(x) + [f'(x)]^2 = 0$ is

(A) f(x) = c . $e^{(2-\sqrt{3})x}$ (B) f(x) = c . $e^{(2+\sqrt{3})x}$

(C) f(x) = c . $e^{(\sqrt{3}-2)x}$ (D) f(x) = c . $e^{-(2+\sqrt{3})x}$

3. The equation of the curve passing through (3, 4) & satisfying the differential equation,

$y\left(\dfrac{dy}{dx}\right)^2 + (x-y)\dfrac{dy}{dx} - x = 0$ can be

(A) x – y + 1=0 (B) $x^2 + y^2$ =25
(C) $x^2 + y^2$ – 5x – 10=0 (D) x + y – 7=0

4. The graph of the function y = f(x) passing through the point (0, 1) and satisfying the differential equation $\dfrac{dy}{dx}$ + y cos x = cos x is such that

(A) it is a constant function
(B) it is periodic
(C) it is neither an even nor an odd function
(D) it is continuous & differentiable for all x.

5. Water is drained from a vertical cylindrical tank by opening a valve at the base of the tank. It is known that the rate at which the water level drops is proportional to the square root of water depth y, where the constant of proportionality k > 0 depends on the acceleration due to gravity and the geometry of the hole. If t is measured in minutes and k = 1/15 then the time to drain the tank if the water is 4 meter deep to start with is

(A) 30 min (B) 45 min (C) 60 min (D) 80 min

6. Number of straight lines which satisfy the differential equation $\dfrac{dy}{dx} + x\left(\dfrac{dy}{dx}\right)^2 - y = 0$ is

(A) 1 (B) 2 (C) 3 (D) 4

7. The solution of the differential equation,

$x^2\dfrac{dy}{dx}.\cos\dfrac{1}{x} - y\sin\dfrac{1}{x} = -1$, where $y \to -1$ as $x \to \infty$ is

(A) y = $\sin\dfrac{1}{x} - \cos\dfrac{1}{x}$ (B) y = $\dfrac{x+1}{x\sin\frac{1}{x}}$

(C) y = $\cos\dfrac{1}{x} + \sin\dfrac{1}{x}$ (D) y = $\dfrac{x+1}{x\cos\frac{1}{x}}$

8. If y = $\dfrac{x}{\ell n|cx|}$ (where c is an arbitrary constant) is the general solution of the differential equation $\dfrac{dy}{dx} = \dfrac{y}{x} + \phi\left(\dfrac{x}{y}\right)$ then the function $\phi\left(\dfrac{x}{y}\right)$ is

(A) $\dfrac{x^2}{y^2}$ (B) $-\dfrac{x^2}{y^2}$ (C) $\dfrac{y^2}{x^2}$ (D) $-\dfrac{y^2}{x^2}$

9. If $\int_a^x t y(t)dt = x^2 + y(x)$ then y as a function of x is

(A) y = 2 – (2 + a^2)$e^{\frac{x^2-a^2}{2}}$ (B) y = 1 – (2 + a^2)$e^{\frac{x^2-a^2}{2}}$

(C) y = 2 – (1 + a^2)$e^{\frac{x^2-a^2}{2}}$ (D) none

10. A function f(x) satisfying $\int_0^1 f(tx)dt = nf(x)$, where x > 0, is

(A) f(x) = $c.x^{\frac{1-n}{n}}$ (B) f(x) = $c.x^{\frac{n}{n-1}}$

(C) f(x) = $c.x^{\frac{1}{n}}$ (D) f(x) = $c.x^{(1-n)}$

11. The differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} + \sin y + x^2 = 0$ is of the following type

(A) linear (B) homogeneous
(C) order two (D) degree one

12. A curve C passes through origin and has the property that at each point (x, y) on it the normal line at that point passes through (1, 0). The equation of a common tangent to the curve C and the parabola $y^2 = 4x$ is

(A) $x = 0$ (B) $y = 0$ (C) $y=x+1$ (D) $x+y+1=0$

13. The solution of the differential equation

$$\left(\frac{dy}{dx}\right)^2 - \frac{dy}{dx}(e^x + e^{-x}) + 1 = 0 \text{ is}$$

(A) $y + e^{-x} = c$ (B) $y - e^{-x} = c$
(C) $y + e^x = c$ (D) $y - e^x = c$

14. Let $y = (A + Bx)e^{3x}$ be a solution of the differential equation $\frac{d^2y}{dx^2} + m\frac{dy}{dx} + ny = 0$, $m, n \in I$, then

(A) $m+n=3$ (B) $n^2 - m^2=64$ (C) $m=-6$ (D) $n=9$

15. The differential equation $2xy\, dy = (x^2 + y^2 + 1)\, dx$ determines

(A) A family of circles with centre on x-axis
(B) A family of circles with centre on y-axis
(C) A family of rectangular hyperbola with centre on x-axis
(D) A family of rectangular hyperbola with centre on y-axis

16. If $f''(x) + f'(x) + f^2(x) = x^2$ be the differential equation of a curve and let P be the point of maxima then number of tangents which can be drawn from point P to $x^2 - y^2 = a^2$ is

(A) 2 (B) 1 (C) 0 (D) either 1 or 2

17. The solution of $x^2dy - y^2dx + xy^2(x - y)dy = 0$ is

(A) $\ell n\left|\frac{x-y}{xy}\right| = \frac{y^2}{2} + c$ (B) $\ell n\left|\frac{xy}{x-y}\right| = \frac{x^2}{2} + c$

(C) $\ell n\left|\frac{x-y}{xy}\right| = \frac{x^2}{2} + c$ (D) $\ell n\left|\frac{x-y}{xy}\right| = x + c$

18. The orthogonal trajectories of the system of curves $\left(\frac{dy}{dx}\right)^2 = \frac{4}{x}$ are

(A) $9(y + c)^2 = x^3$ (B) $y + c = \frac{-x^{3/2}}{3}$

(C) $y + c = \frac{x^{3/2}}{3}$ (D) all of these